



**Semester One Examination, 2022**

**Question/Answer booklet**

**MATHEMATICS  
SPECIALIST  
UNIT 1**

**Section Two:  
Calculator-assumed**

Your name \_\_\_\_\_

Your Teacher's Name \_\_\_\_\_

**Time allowed for this section**

Reading time before commencing work: ten minutes  
Working time: one hundred minutes

**Materials required/recommended for this section**

***To be provided by the supervisor***

This Question/Answer booklet  
Formula sheet (retained from Section One)

***To be provided by the candidate***

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators approved for use in this examination

**Important note to candidates**

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Question	Maximum	Marks	Question	Maximum	Marks
7	8		13	7	
8	6		14	9	
9	9		15	8	
10	11		16	9	
11	8		17	10	
12	8				

## Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	6	6	50	47	35
Section Two: Calculator-assumed	11	11	100	93	65
<b>Total</b>					100

## Instructions to candidates

1. The rules for the conduct of examinations are detailed in the school handbook. Sitting this examination implies that you agree to abide by these rules.
2. Write your answers in this Question/Answer booklet preferably using a blue/black pen.
3. You must be careful to confine your answers to the specific question asked and to follow any instructions that are specific to a particular question.
4. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.
5. It is recommended that you do not use pencil, except in diagrams.
6. Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.
7. The Formula sheet is not to be handed in with your Question/Answer booklet.

**Section Two: Calculator-assumed****65% (93 Marks)**

This section has **eleven** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time: 100 minutes.

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**Question 7****(8 marks)**

(a) Determine the number of ways that 13 persons can be seated in a row of 6 seats.

**(2 marks)**

(b) A five-digit number divisible by 3 is to be formed using digits 0, 1, 2, 3, 4 and 5 without repetition. Determine the total number of ways this can be done.

**(3 marks)**

(c) There are 10 points in a plane, out of which 4 are collinear. Determine the number of triangles made by these 10 points.

**(3 marks)**

**Question 8****(6 marks)**

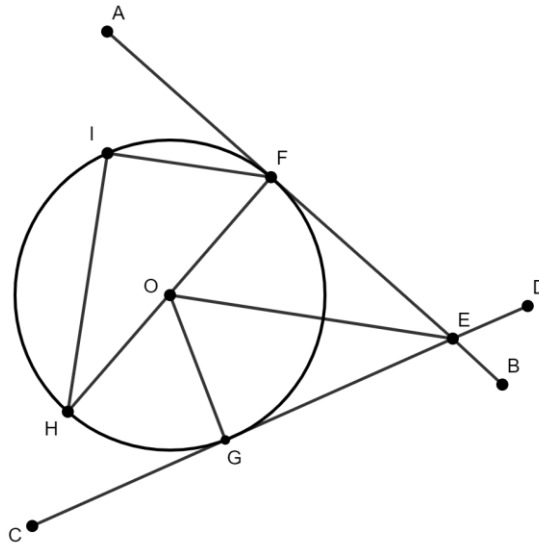
- (a) In a group of 20 people, friendship is mutual. Use Pigeonhole principle to show that there exist two people who have the same number of friends. (3 marks)

- (b) A box contains 6 red, 8 green, 10 blue, 12 yellow and 15 white marbles. Determine the minimum number of marbles we must choose randomly from the box to ensure that we get 9 marbles of same colour. (3 marks)

**Question 9**

**(9 marks)**

The diagram below (not to scale) showing a circle with centre at O and diameter HF. Point I lies on the circumference. AB and CD are tangents to the circle at Points F and G respectively. Point E is the intersection of AB and CD. Given  $\angle IHF = 31^\circ$  and  $\angle DEB = 78^\circ$ , determine the size of each angle in the table, giving reason(s).



	Size	Reason(s)
$\angle HIF$		
$\angle AFI$		
$\angle FOG$		
$\angle OEF$		

## Question 10

(11 marks)

- (a) Draw two non-zero, non-parallel vectors,  $\mathbf{a}$  and  $\mathbf{b}$ .

Show with a diagram that

$$\mathbf{a} + \mathbf{b} = \mathbf{b} + \mathbf{a} \quad (2 \text{ marks})$$

- (b) Two vectors are given as follows:  $\mathbf{a} = 7\mathbf{i} + y\mathbf{j}$ , and  $\mathbf{b} = 18\mathbf{i} + 4\mathbf{j}$

- (i) Find a value for  $y$  such that  $\mathbf{a}$  and  $\mathbf{b}$  are parallel. (3 marks)

- (ii) Find a value for  $y$  such that  $\mathbf{a}$  and  $\mathbf{b}$  are perpendicular. (3 marks)

- (iii) Find the value(s) of  $y$  if the angle between  $\mathbf{a}$  and  $\mathbf{b}$  is  $60^\circ$ . (3 marks)

## Question 11

(8 marks)

(a) Prove that  $7.\overline{53}$  is a rational number.

(4 marks)

(b) Let  $p$  be an irrational number and  $q$  a rational number. Use the method of proof by contradiction to prove that  $p - q$  is irrational. (4 marks)

**Question 12****(8 marks)**

5 letter "words" are chosen using the letters from the word DISCOVERY where each letter can only be used once.

- (a) How many different words are possible? (2 marks)
- (b) How many different words are possible which have exactly 2 vowels? (2 marks)
- (c) How many different words are possible that have the letters E, R and Y next to each other (in any order)? (2 marks)
- (d) How many different words start with a D and end in a Y? (2 marks)



## Question 13

(7 marks)

(a) Using  ${}^n\mathbf{C}_r = \frac{n!}{r!(n-r)!}$ , prove that  ${}^n\mathbf{C}_k = {}^{n-1}\mathbf{C}_k + {}^{n-1}\mathbf{C}_{k-1}$

(4 marks)

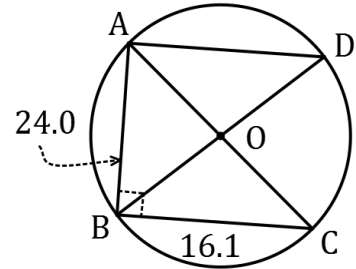
(b) Find the positive integer  $k$  if  ${}^k\mathbf{C}_{k-4} = 2^{k-1}\mathbf{C}_4$ .

(3 marks)

## Question 14

(9 marks)

- (a) Points  $A, B, C$  and  $D$  lie in order on the circumference of the circle with centre  $O$  so that  $AB = 24.0$  cm,  $BC = 16.1$  cm, and  $AC$  and  $BD$  are diameters. Determine, with brief reasons and to the nearest degree, the sizes of  $\angle ACB$ ,  $\angle ADB$ ,  $\angle AOB$  and  $\angle ABD$ . (5 marks)



- (b) Points  $P, Q$  and  $R$  lie on the circumference of a circle of radius 11.7 cm, so that  $PR = 10.3$  cm and  $QR = 20.8$  cm. Prove by contradiction that the midpoint of chord  $PQ$  is not the centre of the circle. (4 marks)

**Question 15****(8 marks)**

Relative to boat  $O$  at anchor in a lake, four buoys  $A, B, C$  and  $D$  have the following position vectors (with distances in metres):

$$\vec{OA} = \begin{pmatrix} -380 \\ -420 \end{pmatrix}, \quad \vec{OB} = \begin{pmatrix} -12 \\ 342 \end{pmatrix}, \quad \vec{OC} = \begin{pmatrix} 420 \\ 550 \end{pmatrix}, \quad \vec{OD} = \begin{pmatrix} 268 \\ -108 \end{pmatrix}.$$

- (a) Prove that the quadrilateral with vertices  $ABCD$  is a trapezium, but not a parallelogram. (5 marks)

- (b) Boat  $X$  motors directly from  $D$  to  $B$  with a constant velocity in 3 minutes and 20 seconds. Determine the velocity in component form, and hence the speed, of boat  $X$ . (3 marks)

## Question 16

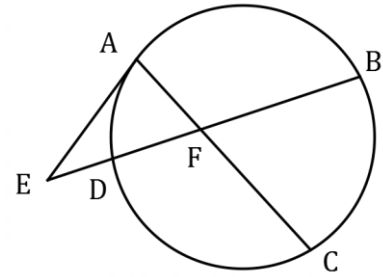
(9 marks)

- (a) In the diagram (not to scale),  $EA$  is a tangent to the circle at  $A$ . Secant  $BE$  cuts chord  $AC$  at  $F$ , and the circle at  $D$ .

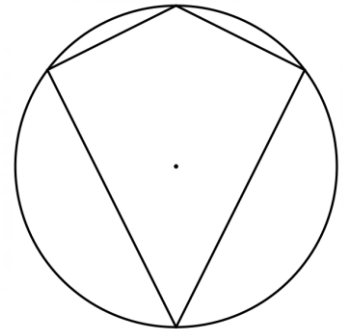
(4 marks)

$DE = 5$  cm,  $DB = 40$  cm,  $AE = FE$ ,  
and  $CF$  is 1 cm longer than  $AE$ .

Determine the length of  $AF$ .



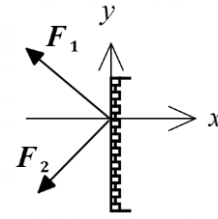
- (b) The vertices of a kite lie on the circumference of a circle. Each longer side of the kite is twice the length of the adjacent shorter side. If the area of the kite is  $18 \text{ cm}^2$ , determine the radius of the circle. (5 marks)



**Question 17**

**(10 marks)**

The diagram at right, not to scale, shows forces  $F_1$  and  $F_2$  acting in the same vertical plane on a small hook fixed to a vertical wall.  $F_1$  has magnitude 147 N and acts at an angle of elevation of  $22^\circ$  and  $F_2$  has magnitude 195 N and acts at an angle of depression of  $42^\circ$ .



The resultant of  $F_1$  and  $F_2$  is  $R$ .

(a) Sketch a triangle to show the relationship between  $F_1$ ,  $F_2$  and  $R$ . (2 marks)

(b) Determine, with reasoning, the magnitude of  $R$  and the acute angle it makes with the wall. (5 marks)

(c) The wall exerts a force on the hook of equal magnitude to  $R$  but in the opposite direction. Express this force using unit vectors  $\mathbf{i}$  and  $\mathbf{j}$ . (3 marks)

Supplementary page

Question number: \_\_\_\_\_

Supplementary page

Question number: \_\_\_\_\_

